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Two-Level Atom in a Standing Electromagnetic Wave

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We study a two-level atom interacting with a standing electromagnetic wave, and work out the wave functions, the energy values and the momentum values of the atom.

KEY WORDS: two-level atom; standing electromagnetic wave.

1. INTRODUCTION

In recent years, the subject of atomic motion in an electromagnetic wave has attracted much attention because of its important application (Arimondo *et al.*, 1981; Cirac *et al.*, 1994; Cook, 1979; Cook *et al.*, 1985; Cook and Bernhardt, 1978; Dalibard and Tannoudji, 1989; Doery *et al.*, 1995; Marte *et al.*, 1004; Mittleman *et al.*, 1977; Stenhdm, 1986; Wineland and Itano, 1979; Yariv, 1989). In this paper, we study the motion of a two-level atom in a standing electromagnetic wave, and work out the wave functions, the energy values and the momentum values of the atom.

2. SCHRÖDINGER'S EQUATION OF A TWO-LEVEL ATOM IN A STANDING ELECTROMAGNETIC WAVE

We consider a two-level atom of mass m, dipole moment $\vec{D}$, which starts out moving in the Z-direction with momentum P_0, then is irradiated by a standing electromagnetic wave with a wave vector k and an angular frequency ω_L. The standing electromagnetic wave propagates along the positive Z-direction, and its

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electronic field $\vec{E}$ is assumed to be the form.

$$\vec{E} = E_x \vec{i} + E_y \vec{j} \tag{1}$$

$$E_x = A \cos kz \cos \omega_L t \tag{2}$$

$$E_y = A \cos kz \cos \omega_L t \tag{3}$$

where A is the amplitude of $\vec{E}$.

The Hamiltonian of a two-level atom interacting with a standing electromagnetic wave is given by

$$H = \frac{p^2}{2m} + \frac{1}{2}\hbar\omega\sigma_3 + V \tag{4}$$

where $\frac{P^2}{2m}$ is the kinetic energy associated with the center-of-mass momentum along the Z direction, $\frac{1}{2}\hbar\omega\sigma_3$ is the Hamiltonian associated with the internal motion of the atom, and $V = -\vec{D} \cdot \vec{E}$ is dipole interacting energy between the atom and the standing wave.

We denote the ground state vector and excited state vector of the two-level atom by $|2\rangle$ and $|1\rangle$. According to the feature of the dipole transition (Yariv, 1989), the action of V on these state vectors are

$$V|1\rangle = \upsilon_{21}|2\rangle \quad \text{and} \quad V|2\rangle = \upsilon_{12}|1\rangle,$$

where

$$\upsilon_{12} = \langle 1|V|2\rangle \quad \text{and} \quad \upsilon_{21} = \langle 2|V|1\rangle$$

are the nonzero matrix elements of V.

Setting $D^{\pm} = D_x \pm iD_y$ and $E^{\pm} = E_x \pm iE_y$, V may be written as

$$V = -\frac{1}{2}(D^+E^- + D^-E^+) \tag{5}$$

Using

$$D_{12}^- = \langle 1|D^-|2\rangle = D_{21}^+ = \langle 2|D^+|1\rangle = 0 \tag{6}$$

we have

$$D_{12}^+ = D_{21}^- = \langle 1|D^+|2\rangle = 2\langle 1|D_x|2\rangle = 2D \tag{7}$$

where

$$D = \langle 1|D_x|2\rangle \tag{8}$$

then we have

$$\upsilon_{12} = -DE^- = -D(E_x - iE_y) \qquad \upsilon_{21} = -DE^+ = -D(E_x + iE_y) \tag{9}$$

using Eqs. (2), (3), and (9), we have

$$\upsilon_{12} = -\hbar\Omega \cos kz \, e^{i\omega_L t} \qquad \upsilon_{21} = -\hbar\Omega \cos kz \, e^{-i\omega_L t} \tag{10}$$

where $\hbar\Omega = 2DA$, Ω is called induced rat, which describes the interaction intensity.

In order to study the motion of the system, we solve the Schrödinger equation

$$i\hbar \frac{d}{dt}|\varphi\rangle = H|\varphi\rangle \tag{11}$$

for an arbitrary state $|\varphi\rangle$.

Expanding the state $|\varphi\rangle$ in terms of $|2\rangle$ and $|1\rangle$, we have

$$|\varphi\rangle = \varphi_1(z,t)|1\rangle + \varphi_2(z,t)|2\rangle \tag{12}$$

using Eqs. (4), (5), (10), and (12), Eq. (11) is reduced to

$$i\hbar \frac{d}{dt}\varphi_1 = \left(\frac{p^2}{2m} + \frac{1}{2}\hbar\omega\right)\varphi_1 - \frac{1}{2}\hbar\Omega \, e^{-i\omega_L t}(e^{ikz} + e^{-ikz})\varphi_2 \tag{13}$$

$$i\hbar \frac{d}{dt}\varphi_2 = \left(\frac{p^2}{2m} - \frac{1}{2}\hbar\omega\right)\varphi_2 - \frac{1}{2}\hbar\Omega \, e^{i\omega_L t}(e^{ikz} + e^{-ikz})\varphi_1 \tag{14}$$

3. SOLUTION OF THE SCHRÖDINGER EQUATION

Eliminating φ_1 from Eqs. (13) and (4), we obtain the equation for φ_2

$$\frac{d^2\varphi_2}{dt^2} + i\frac{1}{\hbar}\left(\frac{P^2}{m} - \hbar\omega_L\right)\frac{d\varphi_2}{dt} + \frac{1}{\hbar^2}\left[\left(\frac{P^2}{2m} - \frac{1}{2}\hbar\omega\right)\left(\hbar\omega_L - \frac{1}{2}\hbar\omega - \frac{P^2}{2m}\right)\right.$$
$$\left. + \frac{\hbar^2\Omega^2}{2} + \frac{\hbar^2\Omega^2}{4}e^{i2kz} + \frac{\hbar^2\Omega^2}{4}e^{-i2kz}\right]\varphi_2 = 0 \tag{15}$$

we expand φ_2 as

$$\varphi_2 = \sum_{n=-\infty}^{\infty} C_n(t)\, e^{i(P_0 + n\hbar k)z/\hbar} \tag{16}$$

where P_0 is the initial center-of-mass momentum of the two-level atom. Substituting Eq. (16) into (15), we have

$$C_n''(t) + i\frac{1}{\hbar}\left[\frac{(p_0 + n\hbar k)^2}{m} - \hbar\omega_L\right]C_n'(t) + \frac{1}{\hbar^2}\left\{\left[\frac{(p_0 + n\hbar k)^2}{2m} - \frac{1}{2}\hbar\omega\right]\right.$$
$$\left.\times\left[\hbar\omega_L - \frac{1}{2}\hbar\omega - \frac{(P_0 + n\hbar k)^2}{2m}\right] + \frac{\hbar^2\Omega^2}{2}\right\}C_n(t) + \frac{\Omega^2}{4}C_{n-2}(t)$$
$$+ \frac{\Omega^2}{4}C_{n+2}(t) = 0 \tag{17}$$

For convenience, we rewrite Eq. (17) as

$$C_n''(t) + ia_1(n)C_n'(t) + a_2(n)C_n(t) + \frac{\Omega^2}{4}C_{n-2}(t) + \frac{\Omega^2}{4}C_{n+2}(t) = 0 \quad (18)$$

where

$$a_1(n) = \frac{1}{\hbar}\left[\frac{(P_0 + n\hbar k)^2}{m} - \hbar\omega_L\right]$$

$$a_2(n) = \frac{1}{\hbar^2}\left\{\left[\frac{(P_0 + n\hbar k)^2}{2m} - \frac{1}{2}\hbar\omega_L\right]\left[\hbar\omega_L - \frac{1}{2}\hbar\omega - \frac{(p_0 + n\hbar k)^2}{2m} + \frac{\hbar^2\Omega^2}{2}\right]\right\}$$

Assuming that the intensity of the standing electromagnetic wave is very weak, when $n \neq 0$, one can find relation

$$|C_n(t)| \ll |C_0(t)| \quad (19)$$

Eq. (18) can be written as

$$C_0''(t) + ia_1(0)C_0'(t) + a_2(0)C_0(t) + \frac{\Omega^2}{4}C_{-2}(t) + \frac{\Omega^2}{4}C_2(t) = 0 \quad (20)$$

$$C_2''(t) + ia_1(2)C_2'(t) + a_2(2)C_2(t) + \frac{\Omega^2}{4}C_0(t) + \frac{\Omega^2}{4}C_4(t) = 0 \quad (21)$$

$$C_{-2}''(t) + ia_1(-2)C_{-2}'(t) + a_2(-2)C_{-2}(t) + \frac{\Omega^2}{4}C_0(t) + \frac{\Omega^2}{4}C_{-4}(t) = 0 \quad (22)$$

$$C_4''(t) + ia_1(4)C_4'(t) + a_2(4)C_4(t) + \frac{\Omega^2}{4}C_2(t) + \frac{\Omega^2}{4}C_6(t) = 0 \quad (23)$$

$$C_{-4}''(t) + ia_1(-4)C_{-4}'(t) + a_2(-4)C_{-4}(t) + \frac{\Omega^2}{4}C_{-6}(t) + \frac{\Omega^2}{4}C_{-2}(t) = 0 \quad (24)$$

using

$$|C_2(t)| \ll |C_0(t)| \quad \text{and} \quad |C_{-2}(t)| \ll |C_0(t)|,$$

we can reduce Eq. (20) to

$$C_0''(t) + ia_1(0)C_0'(t) + a_2(0)C_0(t) = 0 \quad (25)$$

solving Eq. (25), we can obtain

$$C_0(t) = A_0\, e^{-iE_2^1 t/\hbar} + B_0\, e^{-iE_2^2 t/\hbar} \quad (26)$$

where

$$E_2^1 = \frac{P_0^2}{2m} - \frac{1}{2}\hbar\omega_L + \frac{\hbar}{2}\sqrt{2\Omega^2 + (\omega - \omega_L)^2} \quad (27)$$

$$E_2^2 = \frac{P_0^2}{2m} - \frac{1}{2}\hbar\omega_L - \frac{\hbar}{2}\sqrt{2\Omega^2 + (\omega - \omega_L)^2} \quad (28)$$

the constants A_0 and B_0 are determined by the initial conditions.

Using

$$|C_4(t)| \ll |C_0(t)| \quad \text{and} \quad |C_{-4}(t)| \ll |C_0(t)|,$$

we reduce Eqs. (21) and (22) to

$$C_2''(t) + ia_1(2)C_2'(t) + a_2(2)C_2(t) + \frac{\Omega^2}{4}C_0(t) = 0 \tag{29}$$

$$C_{-2}''(t) + ia_1(-2)C_{-2}'(t) + a_2(-2)C_{-2}(t) + \frac{\Omega^2}{4}C_0(t) = 0 \tag{30}$$

substituting Eq. (26) into Eqs. (29) and (30), then solving Eqs. (29) and (30), we have

$$C_2(t) = A_2\, e^{-iE_2^3t/\hbar} + B_2\, e^{-iE_2^4t/\hbar} - \frac{\Omega^2 A_0\, e^{-iE_2^1t/\hbar}}{4\left[-\frac{\left(E_2^1\right)^2}{\hbar^2} - a_1(2)\frac{E_2^1}{\hbar} + a_2(2)\right]} - \frac{\Omega^2 B_0\, e^{-iE_2^2t/\hbar}}{4\left[-\frac{\left(E_2^2\right)^2}{\hbar^2} - a_1(2)\frac{E_2^2}{\hbar} + a_2(2)\right]} \tag{31}$$

$$E_2^3 = \frac{(p_0 + 2\hbar k)^2}{2m} - \frac{1}{2}\hbar\omega_L + \frac{1}{2}\hbar\sqrt{2\Omega^2 + (\omega - \omega_L)^2} \tag{32}$$

$$E_2^4 = \frac{(p_0 + 2\hbar k)^2}{2m} - \frac{1}{2}\hbar\omega_L - \frac{1}{2}\hbar\sqrt{2\Omega^2 + (\omega - \omega_L)^2} \tag{33}$$

$$C_{-2}(t) = A_{-2}\, e^{-iE_2^5t/\hbar} + B_{-2}\, e^{-iE_2^6t/\hbar} - \frac{\Omega^2 A_0\, e^{-iE_2^1t/\hbar}}{4\left[-\frac{\left(E_2^1\right)^2}{\hbar^2} - a_1(-2)\frac{E_2^1}{\hbar} + a_2(-2)\right]} - \frac{\Omega^2 B_0\, e^{-iE_2^2t/\hbar}}{4\left[-\frac{\left(E_2^2\right)^2}{\hbar^2} - a_1(-2)\frac{E_2^2}{\hbar} + a_2(-2)\right]} \tag{34}$$

$$E_2^5 = \frac{(p_0 - 2\hbar k)^2}{2m} - \frac{1}{2}\hbar\omega_L + \frac{1}{2}\hbar\sqrt{2\Omega^2 + (\omega - \omega_L)^2} \tag{35}$$

$$E_2^6 = \frac{(p_0 + 2\hbar k)^2}{2m} - \frac{1}{2}\hbar\omega_L - \frac{1}{2}\hbar\sqrt{2\Omega^2 + (\omega - \omega_L)^2} \tag{36}$$

Using

$$|C_6(t)| \ll |C_2(t)| \quad \text{and} \quad |C_{-6}(t)| \ll |C_{-2}(t)|,$$

Eqs. (23) and (24) can be reduced to

$$C_4''(t) + ia_1(4)C_4'(t) + a_2(4)C_4(t) + \frac{\Omega^2}{4}C_2(t) = 0 \tag{37}$$

$$C_{-4}''(t) + ia_1(-4)C_{-4}'(t) + a_2(-4)C_{-4}(t) + \frac{\Omega^2}{4}C_{-2}(t) = 0 \tag{38}$$

substituting Eqs. (31) and (34) into (37) and (38), then solving Eqs. (37) and (38), we have

$$\begin{aligned} C_4(t) = {} & A_4\, e^{-iE_2^7 t/\hbar} + B_4\, e^{-iE_2^8 t/\hbar} - \frac{\Omega^2 A_2\, e^{-iE_2^3 t/\hbar}}{4\left[-\frac{(E_2^3)^2}{\hbar^2} - a_1(4)\frac{E_2^3}{\hbar} + a_2(4)\right]} \\ & - \frac{\Omega^2 B_2\, e^{-iE_2^4 t/\hbar}}{4\left[-\frac{(E_2^4)^2}{\hbar^2} - a_1(4)\frac{E_2^4}{\hbar} + a_2(4)\right]} \\ & + \frac{\Omega^4 A_0\, e^{-iE_2^1 t/\hbar}}{16\left[-\frac{(E_2^1)^2}{\hbar^2} - a_1(4)\frac{E_2^1}{\hbar} + a_2(4)\right]\left[-\frac{(E_2^2)^2}{\hbar^2} - a_1(2)\frac{E_2^2}{\hbar} + a_2(2)\right]} \\ & + \frac{\Omega^4 B_0\, e^{-iE_2^2 t/\hbar}}{16\left[-\frac{(E_2^2)^2}{\hbar^2} - a_1(4)\frac{E_2^2}{\hbar} + a_2(4)\right]\left[-\frac{(E_2^2)^2}{\hbar^2} - a_1(2)\frac{E_2^2}{\hbar} + a_2(2)\right]} \end{aligned} \tag{39}$$

$$E_2^7 = \frac{(p_0 + 4\hbar k)^2}{2m} - \frac{1}{2}\hbar\omega_L + \frac{\hbar}{2}\sqrt{2\Omega^2 + (\omega - \omega_L)^2} \tag{40}$$

$$E_2^8 = \frac{(p_0 + 4\hbar k)^2}{2m} - \frac{1}{2}\hbar\omega_L - \frac{\hbar}{2}\sqrt{2\Omega^2 + (\omega - \omega_L)^2} \tag{41}$$

$$\begin{aligned} C_{-4}(t) = {} & A_{-4}\, e^{-iE_2^9 t/\hbar} + B_{-4}\, e^{-iE_2^{10} t/\hbar} \\ & - \frac{\Omega^2 A_{-2}\, e^{-iE_2^5 t/\hbar}}{4\left[-\frac{(E_2^5)^2}{\hbar^2} - a_1(4)\frac{E_2^5}{\hbar} + a_4(-4)\right]} - \frac{\Omega^2 B_{-2}\, e^{-iE_2^6 t/\hbar}}{4\left[-\frac{(E_2^6)^2}{\hbar^2} - a_1(-4)\frac{E_2^6}{\hbar} + a_2(-4)\right]} \\ & + \frac{\Omega^4 A_0\, e^{-iE_2^1 t/\hbar}}{16\left[-\frac{(E_2^1)^2}{\hbar^2} - a_1(-4)\frac{E_2^1}{\hbar} + a_2(-4)\right]\left[-\frac{(E_2^1)^2}{\hbar^2} - a_1(-2)\frac{E_2^1}{\hbar} + a_2(-2)\right]} \end{aligned}$$

$$+\frac{\Omega^4 B_0\, e^{-iE_2^2 t/\hbar}}{16\left[-\frac{(E_2^2)^2}{\hbar^2}-a_1(-4)\frac{E_2^2}{\hbar}+a_2(-4)\right]\left[-\frac{(E_2^2)^2}{\hbar^2}-a_1(-2)\frac{E_2^2}{\hbar}+a_2(-2)\right]} \tag{42}$$

$$E_2^9=\frac{(p_0-4\hbar k)^2}{2m}-\frac{1}{2}\hbar\omega_L+\frac{\hbar}{2}\sqrt{2\Omega^2+(\omega-\omega_L)^2} \tag{43}$$

$$E_2^{10}=\frac{(p_0-4\hbar k)^2}{2m}-\frac{1}{2}\hbar\omega_L-\frac{\hbar}{2}\sqrt{2\Omega^2+(\omega-\omega_L)^2} \tag{44}$$

where A_0, B_0, A_2, B_2, A_4, B_4, A_{-2}, B_{-2}, A_{-4}, and B_{-4} are determined by the initial conditions.

In terms of Eqs. (27), (28), (32), (33), (35), (36), (40), (41), (43), and (44), it is obtained easily that the ground state energy level

$$E_2=\frac{P_0^2}{2m}-\frac{1}{2}\hbar\omega \tag{45}$$

of the two-level atom is split into E_2^1, E_2^2, E_2^3, E_2^4, E_2^5, E_2^6, E_2^7, E_2^8, E_2^9, $E_2^{10}, \ldots,$ E_2^1 and E_2^2 are the ground state energy levels of the two-level atom that its center-of-mass momentum is P_0, E_2^3 and E_2^4 are the ground state energy levels of the two-levels atom that its center-of-mass moment is $(P_0+2\hbar k)$, E_2^5 and E_2^6 are the ground state energy levels of the two-level atom that its center-of-mass moments is $(P_0-2\hbar k)$, E_2^7 and E_2^8 are the ground state energy levels of the two-level atom that its center-of-mass moments is $(P_0+4\hbar k)$, E_2^9 and E_2^{10} are ground state energy levels of the two-level atom that its center-of-mass moments is $(P_0-4\hbar k)$, and so on. When the two-level atom interact with a standing electromagnetic wave, its center-of-mass momentum is quantization, the only possible eigenvalues of its center-of-mass momentum are $(P_0\pm 2j\hbar k)$, where $j=1,2,3,\ldots$ This result can explain Bragg scattering phenomenon. It will be applied in quantum computation.

Eliminating φ_2 from Eqs. (13) and (4), we obtain the equation for φ_1

$$\frac{d^2\varphi_1}{dt^2}+i\frac{1}{\hbar}\left(\frac{P^2}{m}+\hbar\omega_L\right)\frac{d\varphi_1}{dt}+\frac{1}{\hbar^2}\left[\left(\frac{P^2}{2m}+\frac{1}{2}\hbar\omega_L\right)\left(-\frac{P^2}{2m}+\frac{1}{2}\hbar\omega-\hbar\omega_L\right)\right.$$
$$\left.+\frac{\hbar^2\Omega^2}{2}+\frac{\hbar^2\Omega^2}{2}e^{i2kz}+\frac{\hbar^2\Omega^2}{4}e^{-i2kz}\right]\varphi_1=0 \tag{46}$$

We expand φ_1 as

$$\varphi_1=\sum_{n=-\infty}^{\infty} b_n(t)\, e^{i(P_0+n\,\hbar k)z/\,\hbar} \tag{47}$$

Substituting Eq. (47) into (46), we have

$$b_n''(t) + i\beta_1(n)b_n'(t) + \beta_2(n)b_n(t) + \frac{\Omega^2}{4}b_{n+2}(t) + \frac{\Omega^2}{4}b_{n-2}(t) = 0 \tag{48}$$

where

$$\beta_1(n) = \frac{1}{\hbar}\left[\frac{(P_0 + n\hbar k)^2}{m} + \hbar\omega_L\right]$$

$$\beta_2(n) = \frac{1}{\hbar^2}\left\{\left[\frac{(P_0 + n\hbar k)^2}{2m} + \frac{1}{2}\hbar\omega_L\right]\left[-\frac{(p_0 + n\hbar k)^2}{2m} + \frac{1}{2}\hbar\omega - \hbar\omega_L\right] + \frac{\hbar^2\Omega^2}{2}\right\}$$

Using the same method solving Eq. (18), we can obtain the solution of Eq. (48)

$$b_0(t) = A_0' e^{-iE_1^1 t/\hbar} + B_0' e^{-iE_1^2 t/\hbar} \tag{49}$$

$$E_1^1 = \frac{P_0^2}{2m} + \frac{1}{2}\hbar\omega_L + \frac{\hbar}{2}\sqrt{2\Omega^2 + (\omega - \omega_L)^2} \tag{50}$$

$$E_1^2 = \frac{P_0^2}{2m} + \frac{1}{2}\hbar\omega_L - \frac{\hbar}{2}\sqrt{2\Omega^2 + (\omega - \omega_L)^2} \tag{51}$$

$$b_2(t) = A_2' e^{-iE_1^3 t/\hbar} + B_2' e^{-iE_1^4 t/\hbar} + A_2'' e^{-iE_1^1 t/\hbar} + B_2'' e^{-iE_1^2 t/\hbar} \tag{52}$$

$$E_1^3 = \frac{(P_0 + 2\hbar k)^2}{2m} + \frac{1}{2}\hbar\omega_L + \frac{\hbar}{2}\sqrt{2\Omega^2 + (\omega - \omega_L)^2} \tag{53}$$

$$E_1^4 = \frac{(P_0 + 2\hbar k)^2}{2m} + \frac{1}{2}\hbar\omega_L - \frac{\hbar}{2}\sqrt{2\Omega^2 + (\omega - \omega_L)^2} \tag{54}$$

$$b_{-2}(t) = A_{-2}' e^{-iE_1^5 t/\hbar} + B_{-2}' e^{-iE_1^6 t/\hbar} + A_{-2}'' e^{-iE_1^1 t/\hbar} + B_{-2}'' e^{-iE_1^2 t/\hbar} \tag{55}$$

$$E_1^5 = \frac{(P_0 - 2\hbar k)^2}{2m} + \frac{1}{2}\hbar\omega_L + \frac{\hbar}{2}\sqrt{2\Omega^2 + (\omega - \omega_L)^2} \tag{56}$$

$$E_1^6 = \frac{(P_0 + 2\hbar k)^2}{2m} + \frac{1}{2}\hbar\omega_L - \frac{\hbar}{2}\sqrt{2\Omega^2 + (\omega - \omega_L)^2} \tag{57}$$

$$\begin{aligned} b_4(t) = {} & A_4' e^{-iE_1^7 t/\hbar} + B_4' e^{-iE_1^8 t/\hbar} + A_4'' e^{-iE_1^1 t/\hbar} + B_4'' e^{-iE_1^2 t/\hbar} \\ & + A_4''' e^{-iE_1^3 t/\hbar} + B_4''' e^{-iE_1^4 t/\hbar} \end{aligned} \tag{58}$$

$$E_1^7 = \frac{(P_0 + 4\hbar k)^2}{2m} + \frac{1}{2}\hbar\omega_L + \frac{\hbar}{2}\sqrt{2\Omega^2 + (\omega + \omega_L)^2} \tag{59}$$

$$E_1^8 = \frac{(P_0 + 4\hbar k)^2}{2m} + \frac{1}{2}\hbar\omega_L - \frac{\hbar}{2}\sqrt{2\Omega^2 + (\omega - \omega_L)^2} \tag{60}$$

$$b_{-4}(t) = A'_{-4}\, e^{-iE_1^9 t/\hbar} + B'_{-4}\, e^{-iE_1^{10} t/\hbar} + A''_{-4}\, e^{-iE_1^1 t/\hbar} + B''_{-4}\, e^{-iE_1^2 t/\hbar}$$
$$+A'''_{-4}\, e^{-iE_1^3 t/\hbar} + B'''_{-4}\, e^{-iE_1^4 t/\hbar} \tag{61}$$

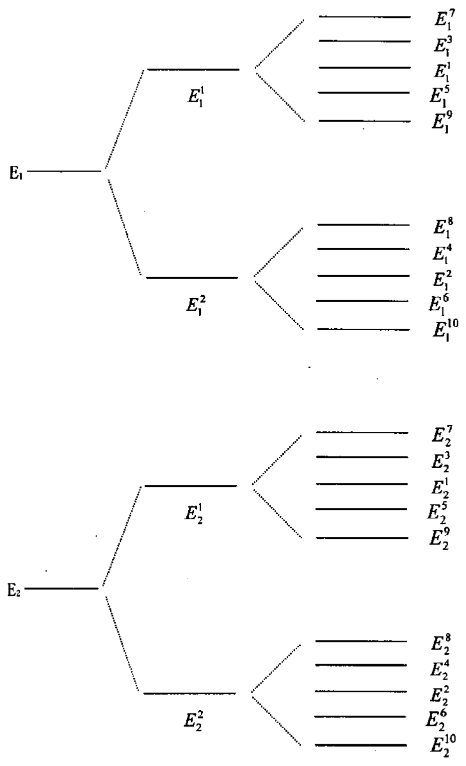
$$E_1^9 = \frac{(P_0 + 4\hbar k)^2}{2m} + \frac{1}{2}\hbar\omega_L + \frac{\hbar}{2}\sqrt{2\Omega^2 + (\omega + \omega_L)^2} \tag{62}$$

$$E_1^{10} = \frac{(P_0 - 4\hbar k)^2}{2m} + \frac{1}{2}\hbar\omega_L - \frac{\hbar}{2}\sqrt{2\Omega^2 + (\omega - \omega_L)^2} \tag{63}$$

... The constants A'_0, B'_0, A'_2, B'_2, A''_2, B''_2, A'_{-2}, B'_{-2}, A''_{-2}, B'_{-2}, ..., are determined by the initial conditions.

4. DISCUSSION

We can draw the energy levels figure by the value of E_2^1, E_2^2, E_2^3, E_2^4, ..., and E_1^1, E_1^2, E_1^3, E_1^4, ...



The figure denote that the ground state energy level E_2 and the excited state energy level E_1 of the two-level atom are split into E_2^1, E_2^2, and E_1^1, E_1^2, due to the coupling between the dipole of the atom and the standing wave, the center-of-mass momentum is changed quantitatively due to the collision between the photon and the atom, the energy levels are split further into $E_2^1, E_2^2, E_2^3, E_2^4, E_2^5, E_2^6, \ldots,$ and $E_1^1, E_1^2, E_1^3, E_1^4, E_1^5, E_1^6, \ldots.$

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